

In a packing plant, a machine packs cartons with jars. It is supposed that a new machine will pack faster on the average than the machine currently used. To test that hypothesis, the times it takes each machine to pack ten cartons are recorded. The results, in seconds, are shown in the following table.

New machine					Old machine				
42.1	41.3	42.4	43.2	41.8	42.7	43.8	42.5	43.1	44.0
41.0	41.8	42.8	42.3	42.7	43.6	43.3	43.5	41.7	44.1
$\bar{x}_1 = 42.14, s_1 = 0.683$					$\bar{x}_2 = 43.23, s_2 = 0.750$				

Do the data provide sufficient evidence to conclude that, on the average, the new machine packs faster? Perform the required hypothesis test at the 5% level of significance.

$H_0: \mu_N - \mu_O = 0$

$H_a: \mu_N - \mu_O < 0$

$\mu_N =$ true mean time to pack new

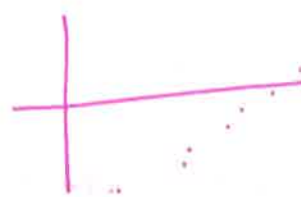
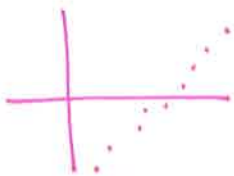
$\mu_O =$ true mean time to pack old

New

old

Random? not given for either. proceed w/ caution

Normal?



Since normal prob plot is \approx linear, OK to assume normality of sample's dist

→ Same here

Independent?

10 (10) ← pop of cartons ✓

10 (10) ← pop of cartons ✓



$$t = \frac{\bar{x}_N - \bar{x}_O}{\sqrt{\frac{s_N^2}{n_N} + \frac{s_O^2}{n_O}}} = -3.397 \quad P = .0016$$

Since $P = .0016 < \alpha = .05$ we reject H_0 . There is evidence that the true mean packing time for the new machine is faster than the mean of the old.

A medical researcher conjectures that smoking can result in wrinkled skin around the eyes. The researcher recruited 150 smokers and 250 nonsmokers to take part in an observational study and found that 95 of the smokers and 105 of the nonsmokers were seen to have prominent wrinkles around the eyes (based on a standardized wrinkle score administered by a person who did not know if the subject smoked or not).

- a) Create a 96% confidence interval to estimate the difference in the proportions of people with prominent wrinkles around the eyes for smokers and nonsmokers.

Smokers

non smokers

Random, no "recruited" proceed w/ caution for both

Normal?

$$150 \left(\frac{95}{150} \right) \geq 10 \quad 150 \left(\frac{55}{150} \right) \geq 10$$

$$250 \left(\frac{105}{250} \right) \geq 10 \quad 250 \left(\frac{145}{250} \right) \geq 10$$

$$10(150) < \text{pop smokers} \quad \checkmark$$

$$10(250) < \text{pop non smokers}$$

$$\hat{p}_S - \hat{p}_N \pm z^* \sqrt{\frac{\hat{p}_S(1-\hat{p}_S)}{n_S} + \frac{\hat{p}_N(1-\hat{p}_N)}{n_N}} = (.110, .316)$$

We are 96% confident that the true difference in proportion of people w/ prominent wrinkles around the eyes for smokers + non smokers is b/w .110 and .316

- b) Based only on this confidence interval, do you think that there is a difference in the proportions of people with wrinkles around the eyes for smokers and nonsmokers? Justify your answer.

Yes. Because 0 is NOT in the 96% confidence interval, thus is evidence that there is a difference in the proportion of people w/ wrinkles around the eyes for smokers + non smokers.

A French study was conducted in the 1990s to compare the effectiveness of using an instrument called a cardiopump with the effectiveness of using traditional cardiopulmonary resuscitation (CPR) in saving lives of heart attack victims. Heart attack patients in participating cities were treated with either a cardiopump or CPR, depending on whether the individual's heart attack occurred on an even-numbered or an odd-numbered day of the month. Before the start of the study, a coin was tossed to determine which treatment, a cardiopump or CPR, was given on the even-numbered days. The other treatment was given on the odd-numbered days. In total, 754 patients were treated with a cardiopump, and 37 survived at least one year; while 746 patients were treated with CPR, and 15 survived at least one year.

(a) The conditions for inference are satisfied in the study. State the conditions and indicate how they are satisfied.

• Random? Yes, random assignment of treatment (coin toss)

Normal? $n\hat{p} \geq 10$ $n(1-\hat{p}) \geq 10$

$754 \left(\frac{37}{754} \right) \geq 10$ $754 \left(\frac{717}{754} \right) \geq 10$

$\hat{p} = \frac{52}{1500}$

Ind? $10n_{cp} = 10(754) < \text{pop}_{cp}$

$n\hat{p} \geq 10$ $n(1-\hat{p}) \geq 10$

$746 \left(\frac{15}{746} \right) \geq 10$ $746 \left(\frac{731}{746} \right) \geq 10$

$10(n_{cpr}) = 7460 < \text{pop}_{cpr}$

(b) Perform a statistical test to determine whether the survival rate for patients treated with a cardiopump is significantly higher than the survival rate for patients treated with CPR.

$H_0: P_A - P_B = 0$ $P_A = P_B$

$H_a: P_A - P_B > 0$ $P_A > P_B$

A = cardiopump

P_A = true prop of patients surviving cardiopump

P_B = true prop of patients surviving CPR

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} \approx 3.066 \rightarrow P = .0011$$

Since $P = .0011$ is $< .05$ we reject H_0 . There is evidence that the survival rate for patients treated w/ cardio. is higher than the " " " " " " CPR.

AP Question (2009 #4)

One of the two fire stations in a certain town responds to calls in the northern half of the town, and the other fire station responds to calls in the southern half of the town. One of the town council members believes that the two fire stations have different mean response times. Response time is measured by the difference between the time an emergency call comes into the fire station and the time the first fire truck arrives at the scene of the fire.

Data were collected to investigate whether the council member's belief is correct. A random sample of 50 calls selected from the northern fire station had a mean response time of 4.3 minutes with a standard deviation of 3.7 minutes. A random sample of 50 calls selected from the southern fire station had a mean response time of 5.3 minutes with a standard deviation of 3.2 minutes.

(a) Construct and interpret a 95 percent confidence interval for the difference in mean response times between the two fire stations.

1 = North 2 = South

$$(\bar{X}_n - \bar{X}_s) \pm t^* \sqrt{\frac{S_n^2}{n_n} + \frac{S_s^2}{n_s}}$$

E { Random? Given "random sample"
 Normal? Since $n = 50$ is large for both, OK to assume normality (CLT)
 Ind? $10(50) = 500 < \#$ of calls for both ✓

E { $(-2.37, .37)$ $df = 96$
 $(-2.4, .4)$ $df = 40$

E { We are 95% confident that the true ~~value~~ difference in means ~~for~~ response time between fire stations is b/w $\underline{\quad}$ + $\underline{\quad}$.

(b) Does the confidence interval in part (a) support the council member's belief that the two fire stations have different mean response times? Explain.

E { Since 0 is within the 95% conf interval there could be no difference. Therefore it does not support the councilman's belief that the 2 fire stations have diff. mean response times.